# REVIEWS

**Hydrodynamic and Hydromagnetic Stability.** By S. CHANDRASEKHAR. Clarendon Press: Oxford University Press, 1961. 652 pp. £5. 5*s*.

This extensive and impressive work is devoted to a number of topics in stability theory, selected principally from those to which the author has made so many contributions, both personally and by the guidance and inspiration he has given to the work of his students. The main important topic omitted in the stability of viscous shear flow—thus the book is complementary to C. C. Lin's monograph.

After a brief introduction, chapters II–V are devoted to the Bénard problem and its modifications when rotation and magnetic fields are taken into account. Chapter VI considers convective instability in spheres and spherical shells, including a study of the effects of rotation. Chapters VII–IX take up the study of Couette flow and other flows between coaxial cylinders, including the hydromagnetic case. Chapter X deals with Rayleigh–Taylor instability, the effects of surface tension, rotation and magnetic fields being considered. Chapter XI is on Kelvin–Helmholtz instability in the inviscid case, taking account of interfacial tension, continuous density stratification, and the effects of rotation and magnetic fields. Chapter XII considers the stability of jets and cylinders held together by gravitational or capillary forces, including the hydromagnetic case. Chapter XIII is on gravitational instability and the effects of rotation and magnetic field on Jeans's criterion. Variational formulae and techniques are emphasized and used throughout the book, and the last chapter considers this topic in a general context. In addition there are several useful appendices.

A typical chapter begins with a general discussion of the problem under consideration, perhaps deriving, and commenting on, some relevant general hydrodynamical theorems. The perturbation equations are then set out in detail, with the boundary conditions and where appropriate a variational formulation. The solutions are next discussed, usually first in a simple case (for instance, free boundary conditions in the Bénard problem) where elementary methods suffice, followed by a presentation of methods which can be used in more difficult cases. Many detailed numerical results of such calculations are given. The possibility that the onset of instability occurs by overstable oscillations rather than steady convection is either proved not to occur or given separate treatment. Finally, any relevant experimental data is presented and discussed, and the chapter closes with extensive bibliographical notes.

The presentation throughout is systematic and thorough and mostly authoritative, though of course some sections are already a bit out of date—an inevitable consequence of writing a book on topics of active current research. The systematic theoretical treatment, the compact presentation of the results of many difficult numerical calculations, the discussion of experimental results and the extensive bibliography make this an extremely useful book for reference purposes—one which will be wanted in the library by all, and on the desk by many, of those whose work is connected with hydrodynamic or hydromagnetic stability.

However, even Homer nods. In spite of the great overall value of this large book, it does contain a few misleading or incorrect statements. It is perhaps relevant to comment briefly on a couple of these, since the book will undoubtedly be extensively used for reference purposes. On page 97 we find the result that the critical temperature gradient for the onset of stationary convection in a rotating layer of fluid heated from below is proportional to $(\Omega/d)^{\frac{4}{3}}\,\kappa\nu^{-\frac{1}{3}}$ for large Taylor number. ($\Omega$ is the angular velocity, $d$ the thickness of the layer, $\kappa$ and $\nu$ the coefficients of thermometric conductivity and kinematic viscosity.) From this the conclusion is drawn that 'an inviscid, ideal fluid, in rotation, is stable with respect to the onset of stationary convection for all adverse temperature gradients. This is clearly a consequence of the Taylor–Proudman theorem.' Now, first of all, the Taylor–Proudman theorem is not immediately relevant to the convection problem, because it is deduced on the assumption that the external force is derivable from a potential, and this is not true of the buoyancy force, whose curl is in fact proportional to $\mathbf{k} \times \nabla T$. However, it can be shown that the usual conclusion about the two-dimensional character of the motion follows anyway. But one must be careful in such limiting processes: if the inviscid limit is taken with fixed Pradtl number instead of fixed $\kappa$, the critical temperature gradient goes to zero instead of infinity and even the slightest adverse temperature gradient would appear to make the fluid unstable. Taking the limit in this way and following the steady convection solutions one finds in fact that for any fixed adverse temperature gradient there is a limiting neutral solution having a certain limiting horizontal wave-number. If the magnitude of the vertical velocity perturbation is held fixed, the temperature and vorticity perturbations become infinite; if they are held finite, the vertical velocity perturbation is zero in the limit, and the flow *is* two-dimensional; nevertheless it is appropriate to call it a 'steady convection'. Furthermore, at all larger horizontal wave-numbers there are unstable solutions having a real growth factor $e^{pt}$, and these are not two-dimensional. If $\kappa$ is held fixed as $\nu \to 0$ the flow is also unstable, by overstable oscillations, for any positive adverse temperature gradient. The point is that the statement quoted above, and perhaps even more the discussion in § 24, seem to imply that even the slightest rotation is capable of stabilizing so fundamentally unstable a situation as an inviscid layer with an adverse temperature gradient, and this is not so. The discussion in §§ 40 ($c$) and 41 of the magnetic analogue of the Taylor–Proudman theorem is similarly misleading: the fact that steady small inviscid flows must be two-dimensional in the presence of a vertical magnetic field does not mean that the inviscid case is necessarily stable; see, for instance, equation IV 169, p. 172.

In the chapter on the stability of Couette flow, a presentation is given (§ 67) of Rayleigh's result that a sufficient condition for stability, *with respect to axisymmetric perturbations*, of inviscid Couette flow is that the square of the circulation should nowhere decrease outwards. This is followed by an alleged proof that Rayleigh's condition is necessary and sufficient for stability of such flows to *arbitrary, three-dimensional perturbations*. While the argument given has some ingenious features, I find it incomprehensible at certain points, and the result is certainly wrong. A simple counter-example is given by a flow with constant angular velocity in, say, the inner half of the region between the cylinders and another, larger, constant angular velocity in the outer half. Rayleigh's criterion is satisfied, but the flow is obviously unstable except to axisymmetric perturbations. In fact, the stability problem is trivial to solve explicitly in the case of two-dimensional perturbations and the flow is actually unstable to all (positive integral) wave-numbers. Other examples, without the discontinuity, can also be readily constructed. Rayleigh himself clearly recognized that the circulation-increasing-outwards condition was restricted to axisymmetric perturbations and gave a different condition (monotonic vorticity) sufficient for stability with respect of two-dimensional perturbations. It should be mentioned that the failure of Rayleigh's circulation criterion for arbitrary perturbations does not affect the illustrative examples given in the book, which are concerned with the inviscid stability of velocity profiles actually realizable as steady viscous flows between rotating cylinders. These have uniform vorticity, and Rayleigh's vorticity stability condition implies their stability to two-dimensional perturbations, and, as the detailed calculation shows, they are also stable to arbitrary perturbations if the first condition is satisfied.

Similar criticisms apply to the arguments of § 78 (*b*) in which Rayleigh's criterion is supposedly extended to the case of axial flow in addition to the rotational flow with circulation increasing outwards. A simple counter-example is a rotational flow with constant angular velocity superposed on an axial flow which is zero in the inner half of the space between the cylinders and a non-zero constant in the outer half.

These criticisms apply to only a very small fraction of the work and by no means affect the conclusion that as a whole this book is a most valuable contribution.

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